Unit 4 Rational Expressions
Section 1 Simplifying Rational Expressions

Objective: Simplify Rational Expressions

Rational Expression:

Excluded Values:

Find any excluded values of each rational expression.

1. $\frac{6}{g+4}$

2. $\frac{y-2}{y(y+5)}$

3. $\frac{x+3}{x^2+5x+4}$

4. $\frac{12}{t+5}$

5. $\frac{3b}{b^2+5b}$

6. $\frac{3k^2}{k^2+7k+12}$

A rational expression is in its **simplest form** when the numerator and denominator have **no common factors except 1**. Remember that to simplify fractions you can divide out common factors that appear in both the numerator and the denominator. You can do the same to simplify rational expressions. **FACTORING MAY BE REQUIRED TO FIND COMMON FACTORS**

Simplify each rational expression, if possible. Identify any excluded values.

7. $\frac{2r^4}{14r}$

8. $\frac{6n^2+3n}{2n+1}$

Caution

Be sure to use the original denominator when finding excluded values. The excluded values may not be "seen" in the simplified denominator.

9. $\frac{w^2-4}{w^2-8w+12}$

10. $\frac{5m^2}{15m}$

11. $\frac{6p^3+12p}{p^2+2}$

12. $\frac{x+2}{x^2+5x+6}$

Recall that opposite binomials can help you factor polynomials. Recognizing opposite binomials can also help you simplify rational expressions.

13. $\frac{-10+5x}{x^2-4}$

14. $\frac{18-6r}{2r^2+8r-42}$

15. $\frac{6-2x}{2x^2-4x-6}$

16. $\frac{m+2}{m^2-3m-10}$

Unit 4 Rational Expressions

Section 2 Multiplying and Dividing Rational Expressions

Objective: Perform multiplication and division with rational expressions.

Multiplying Rational Expressions
1. Factor all numerators and denominators completely.
2. Divide out common factors of the numerators and denominators.
3. Multiply numerators. Then multiply denominators.
4. Be sure the numerator and denominator have no common factors other than 1.

Multiply. Simplify your answer.

1. $\frac{6r^2}{5s^3} * \frac{3r^2}{7s}$

2. $\frac{8x^2y^5}{5yz^2} * \frac{10x^2z^2}{16y^3}$

3. $\frac{c-1}{2} * \frac{4}{3c-3}$

4. $\frac{c-4}{5} * \frac{45}{-4c+16}$

Remember!

Just as you can write an integer as a fraction, you can write any expression as a rational expression by writing it with a denominator of 1.

Multiply. Simplify your answer.

5. $(x^2 - 6x + 9) * \frac{2x}{6x-18}$

6. $\frac{4p}{8p+16} * (p^2 - 5p - 14)$

7. $\frac{3a^2+6a}{12b^2} * \frac{2b^3}{3ab+6b}$

8. $\frac{n-5}{n^2+4n} * \frac{n^2+8n+16}{n^2-3n-10}$

9. $\frac{p+4}{p^2+2p} * \frac{p^2-3p-10}{p^2-16}$

10. $\frac{a^2+6ab}{b} * \frac{5+3a}{3a^2b+5ab}$

The rules for dividing rational expressions are the same as the rules for dividing fractions. To divide by a rational expression, multiply by its **reciprocal.**

Divide. Simplify your answer.

11. $\frac{5x^4}{8x^2y^2} \div \frac{15}{8y^5}$

12. $\frac{x^2}{4} \div \frac{x^4y}{12y^2}$

13. $\frac{2}{g} \div \frac{g+4}{g^2}$

14. $\frac{n^2-1}{n} \div \frac{n-1}{n^2-4n}$

15. $\frac{x^4-9x^2}{x^2-4x+3} \div \frac{x^4+2x^3-8x^2}{x^2-16}$

16. $\frac{2x^2-7x-4}{x^2-9} \div \frac{4x^2-1}{8x^2-28x+12}$

17. $\frac{x^2+4x+3}{x^2-4} \div \frac{x^2+2x-3}{x^2-6x+8}$

18. $\frac{x^2-x}{x+2} \div (x^2+2x-3)$

Unit 4 Rational Expressions
Section 3 Adding and Subtracting Rational Expressions

Objective: Perform addition and subtraction with rational expressions.

Adding and subtracting rational expressions is similar to adding and subtracting fractions. To add or subtract rational expressions with like denominators, add or subtract the numerators and use the same denominator.

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5} \qquad \frac{6}{7} - \frac{4}{7} = \frac{2}{7}$$

Add or Subtract the Rational Expressions

1. $\frac{4}{3k^2} + \frac{k}{3k^2}$

2. $\frac{3}{k} - \frac{k+2}{k}$

Caution

Make sure you add the opposite of all the terms in the numerator of the second expression when subtracting rational expressions.

3. $\frac{3y^2}{y+1} + \frac{3y}{y+1}$

4. $\frac{3x-4}{x^2+1} - \frac{6x+1}{x^2+1}$

5. $\frac{3x^2-5}{3x-1} + \frac{2x^2-3x-2}{3x-1}$

6. $\frac{m^2+2m}{m+4} + \frac{3m+4}{m+4}$

To add or subtract rational expressions with unlike denominators, first find the least common denominator (LCD). The LCD is the least common multiple of the polynomials in the denominators.

Least Common Multiple (LCM) of Polynomials
To find the LCM of polynomials:
1. Factor each polynomial completely. Write any repeated factors as powers. For example, $x^3 + 6x^2 + 9x = x(x+3)^2$.
2. List the different factors. If the polynomials have common factors, use the highest power of each common factor.

Find the least common multiple for each pair.

7. $4x^2y^3$ and $6x^4y^5$

8. $4x^3y^7$ and $3x^5y^4$

9. $x^2 - 2x - 3$ and $x^2 - x - 6$

10. $x^2 - 4$ and $x^2 + 5x + 6$

11. $x^2 - 6x + 5$ and $x^2 + x - 2$

12. $x^2 - 25$ and $x^2 + 10x + 25$

To add or subtract rational expressions with unlike denominators, rewrite both expressions with the LCD. This process is similar to adding and subtracting fractions.

Add or Subtract the Rational Expressions

13. $\frac{2x}{x+3}+\frac{3x}{x-2}$

14. $\frac{3x-2}{2x+5}-\frac{2}{5x-2}$

15. $\frac{x}{x+3}+\frac{2x+6}{x^2+6x+9}$

16. $\frac{x}{x+2}-\frac{8}{x^2-4}$

17. $\frac{x-3}{x^2+3x-4}+\frac{2x}{x+4}$

18. $\frac{2x^2+64}{x^2-64}-\frac{x-4}{x+8}$

Unit 4 Rational Expressions
Section 4 Solving Rational Equations

Objective: Solve a rational equation which contains extraneous solutions.

A ____________________________is an equation that contains one or more rational expressions.

If a rational equation is a proportion, it can be solved using the ____________________________.

Solve

1. $\frac{5}{x-2} = \frac{3}{x}$

2. $\frac{4}{h+1} = \frac{2}{h}$

3. $\frac{a}{a+5} = \frac{a-6}{a}$

4. $\frac{x-3}{x} = \frac{x-4}{x-2}$

Some rational equations contain sums or differences of rational expressions. To solve these, you must **find the LCD** of all the rational expressions in the equation and multiply it throughout all parts of the problem.

Solve

5. $\frac{3}{t} - \frac{1}{3t} = \frac{2}{3}$

6. $\frac{x}{x+3} + \frac{1}{x-3} = 1$

7. $\frac{3}{x+1} - \frac{1}{x-2} = \frac{1}{x^2-x-2}$

8. $\frac{6}{x} - \frac{2}{x-1} = 1$

9. $\frac{1}{y-2} + \frac{1}{y+2} = \frac{4}{y^2-4}$

10. $\frac{1}{x+1} + \frac{x}{x-3} = \frac{12}{x^2-2x-3}$

Let's go back and check the answers we got in 9 and 10. What do we notice happens?

These are called ________________________________, a solution of an equation derived from an original equation that is not a solution of the original equation. When you solve a rational equation, it is possible to get extraneous solutions. These values should be eliminated from the solution set. Always check your solutions by substituting them into the original equation.

Solve. Check for Extraneous Roots

11. $\frac{3}{x-7} = \frac{x-2}{x-7}$

12. $\frac{x}{x-2} + \frac{30}{x+2} = 9$

13. $\frac{5}{x^2+x-6} = 2 - \frac{x-3}{x-2}$

14. $\frac{x}{x+3} + \frac{1}{x-1} = \frac{4}{x^2+2x-3}$

15. $\frac{2}{a-1} - \frac{a}{a+3} = \frac{6}{a^2+2a-3}$

16. $\frac{x}{x-2} - \frac{2}{x+3} = \frac{10}{x^2+x-6}$

Unit 4 Rational Expressions
Section 5 Solving Application Problems involving Rational Equations

Objective: Write and Solve rational equations given an application situation

I. Working Together

Pump A can empty a tank in 9 h and pump B can empty it in 15 h. When both pumps were used, how long will they take to empty the tank?

$$\frac{h}{9}+\frac{h}{15}=1$$

Tommy and Susie can paint a room in 4 h and 6 h, respectively. Tommy begins at noon and then Susie begins helping at 2 PM. At what time was the entire room painted?

$$\frac{h}{4}+\frac{h-2}{6}=1$$

Pipe A can fill a tank with water in 4 hours. Pipe B can fill the same tank in 5 hours.
How long will it take both pipes working together to fill the tank?

One crew can detassel a field of corn in 6 days. Another crew can do the same job in 4 days. If the slower crew workers alone for the first two days of detasseling and then is joined by the faster crew, how long will it take the two crews to finish?

The intake pipe can fill a certain tank in 6 h when the outlet pipe is closed, but with the outlet pipe open it takes 9 h. How long would it take the outlet pipe to empty a full tank?

II. Member Rate

Members of the Ski Club contributed equally to obtain \$3000 for a holiday trip. When 8 members found that they could not go, their contributions were refunded and each remaining member than had to pay \$25 more to raise the \$3000. How many went on the trip?

$$\# \, members \cdot member\ rate = total$$

$$\frac{3000}{n} + 25 = \frac{3000}{n-8}$$

Members of the Foreign Language Club contributed equally to obtain \$1800 for a holiday trip. When 6 members found that they could not go, their contributions were refunded and each remaining member then had to pay \$10 more to raise the \$1800. How many went on the trip?

Members of the Computer Club were assessed equal amounts to raise \$1200 to buy some software. When 8 new members joined the group, the assessment was reduced by \$7.50. What was the new size of the club?

III. Traveling

A person bikes to Starved Rock at 12 m/h, stops for 1 h of sightseeing, and rides back at 15 m/h. How far did he travel if the total time for the trip, including the stop, was 7 h? (Look at equivalent forms of uniform motion equation: $d = rt$)

$$\frac{d}{12} + 1 + \frac{d}{15} = 7$$

Steve drives to the Bears game at a rate of 70 m/h, spends 5 hours at the game and drives home at a rate of 60 m/h due to traffic. How far did he travel if the total time of the trip is 9 hours? Round answers to nearest mile.

Brad is boating down river at a rate of 9 m/h, he stops and eats lunch for an hour and then goes back up river at a rate of 6 m/h due to the current. How far did he travel if his entire trip took 6 hours?

IV. Average Rate

A women drove halfway to work at 30 mi/h and the rest of the way at 50 mi/h. What was her average speed for the whole trip? (Hint: Let the distance for the whole trip be, say, 100 mi.)

	Distance (mi)	Rate (mph)	Time (h)
First Half	50	30	$\frac{50}{30}=\frac{5}{3}$
Second Half	50	50	$\frac{50}{50}=1$
Entire Trip	100	r	$\frac{100}{r}$

$$\frac{5}{3}+1=\frac{100}{r}$$

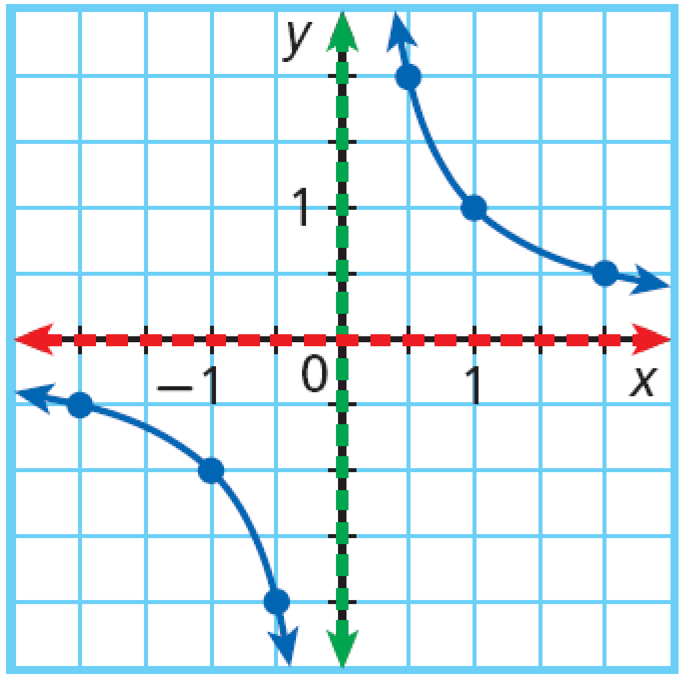
Shawn drove halfway from Peru to Rockford at 40 mi/h and the rest of the way at 60 mi/h. What was his average speed for the whole trip?

A train averaged 80 km/h for the first half of its trip. How fast must it travel for the second half of the trip in order to average 96 km/h for the whole trip?

Unit 4 Rational Expressions
Section 6 Transforming and Graphing Rational Equations

Objective: Identify transformations applied to the rational parent given an equations. Graph a rational function limited to a constant divided by a linear expression using transformations. Identify domain and range of rational functions.

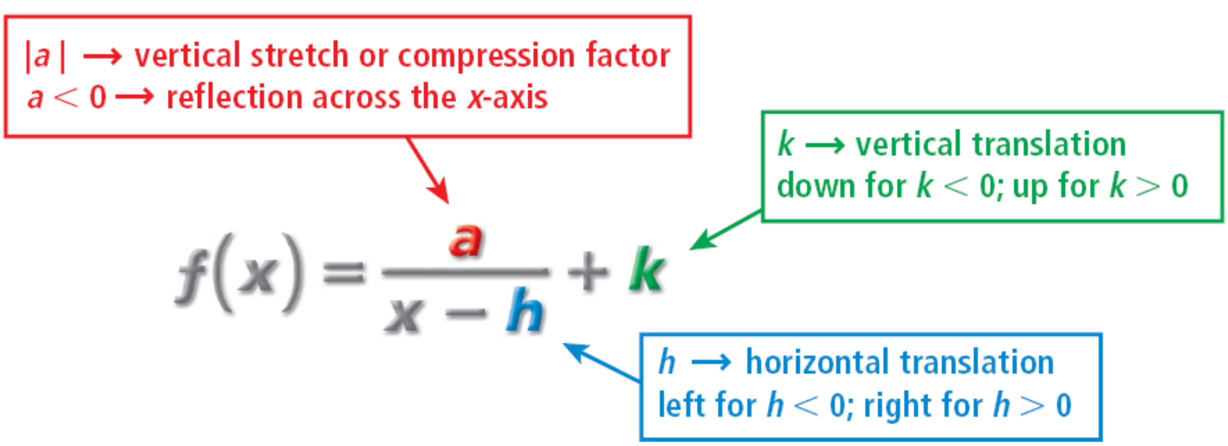
A **rational function** is a function whose rule can be written as a ratio of two polynomials. The parent rational function is $f(x) = \frac{1}{x}$.



Notice the *x*-values as the graph of the function gets closer and closer to the *x*-axis. The function never reaches the *x*-axis because the value of $\frac{1}{x}$ cannot be zero. The same occurs with the y-axis. In this case, the *x*-axis and y-axis are called ***asymptotes***. An **asymptote** is a line that a graphed function approaches but never reaches as the value of ***x or y*** gets very large or very small.

The function $f(x) = \frac{1}{x}$ has a **vertical asymptote at *x* = 0** and a **horizontal asymptote at *y* = 0**.

The rational function $f(x) = \frac{1}{x}$ can be transformed by using methods similar to those used to transform other types of functions.



Describe the transformation of each function.

1. $f(x) = \frac{1}{x-4}$

2. $f(x) = \frac{-1}{x}$

3. $f(x) = \frac{3}{x}$

4. $f(x) = \frac{1}{x} + 3$

5. $f(x) = \frac{2}{x+3}$

6. $f(x) = \frac{-1}{x} - 6$

7. $f(x) = \frac{4}{x-3} + 2$

8. $f(x) = \frac{-2}{x+2} - 3$

The values of *h* and *k* affect the locations of the asymptotes, the domain, and the range of rational functions whose graphs are hyperbolas.

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Rational Functions

For a rational function of the form $f(x) = \dfrac{a}{x - h} + k$,

- the graph is a hyperbola.
- there is a vertical asymptote at the line $x = h$, and the domain is $\{x \mid x \neq h\}$.
- there is a horizontal asymptote at the line $y = k$, and the range is $\{y \mid y \neq k\}$.

Using Transformations Graph the functions. Then identify the asymptotes, domain, and range of the function.

9. $f(x) = \frac{1}{x-1}$

Vertical Asymptote______________

Horizontal Asymptote____________

Domain____________

Range__________

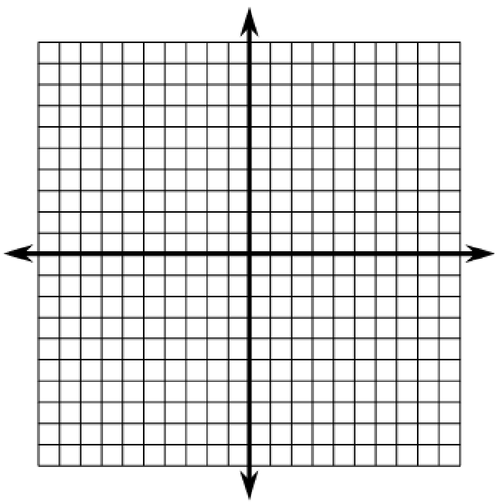
10. $f(x) = \frac{1}{x} + 1$

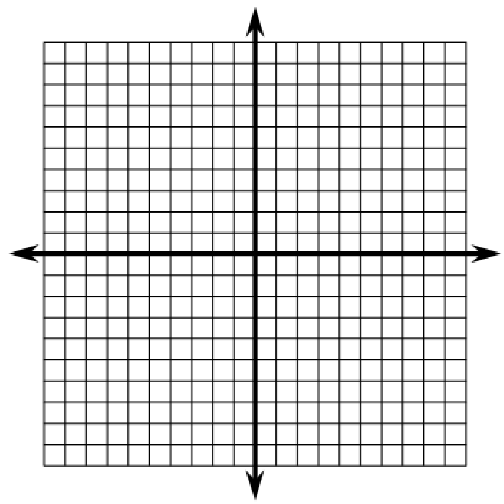
Vertical Asymptote______________

Horizontal Asymptote____________

Domain____________

Range__________





11. $f(x) = -\frac{1}{x}$

Vertical Asymptote______________

Horizontal Asymptote____________

Domain____________

Range__________

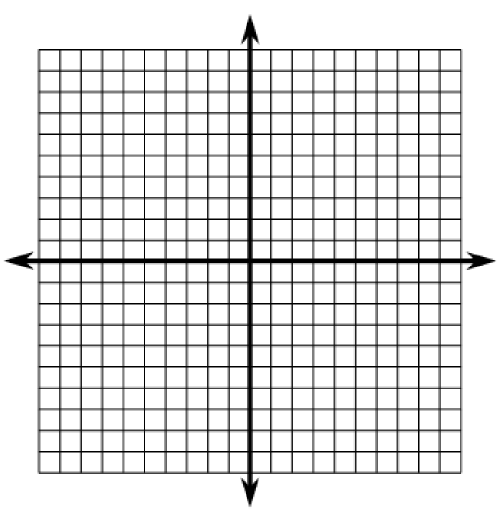
12. $f(x) = \frac{2}{x}$

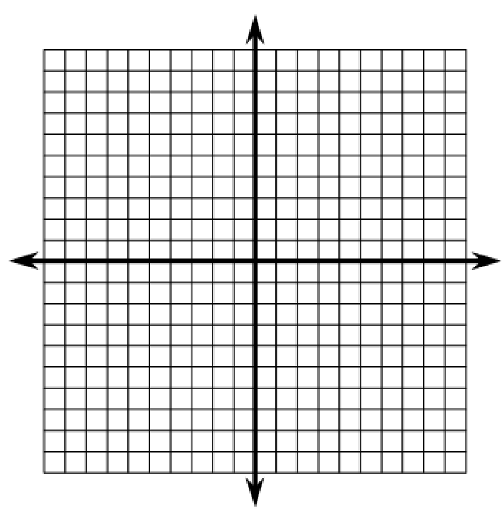
Vertical Asymptote______________

Horizontal Asymptote____________

Domain____________

Range_________





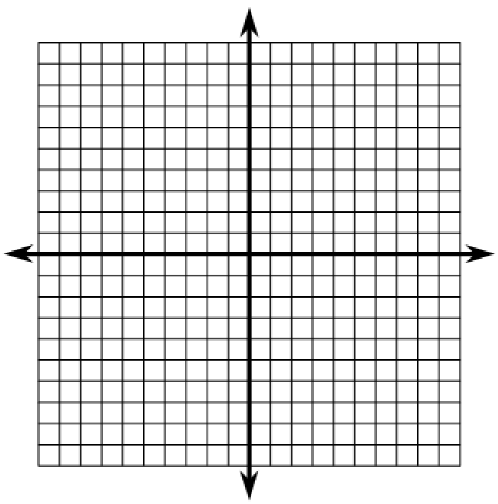
13. $f(x) = \frac{-1}{x+3}$

Vertical Asymptote______________

Horizontal Asymptote____________

Domain____________

Range__________



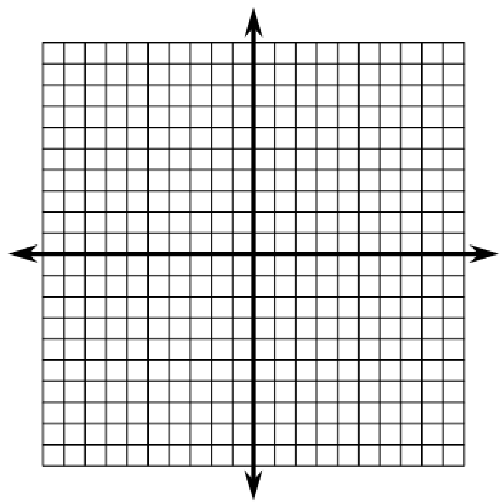
14. $f(x) = \frac{1}{x+2} - 1$

Vertical Asymptote______________

Horizontal Asymptote____________

Domain____________

Range__________



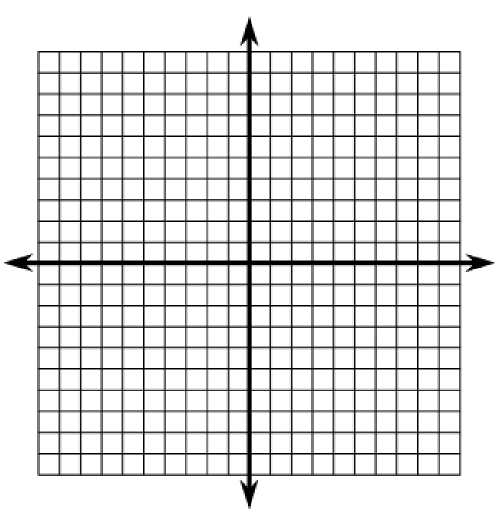
15. $f(x) = \frac{3}{x-2} - 2$

Vertical Asymptote______________

Horizontal Asymptote____________

Domain____________

Range__________



16. $f(x) = \frac{-2}{x-6} + 4$

Vertical Asymptote______________

Horizontal Asymptote____________

Domain____________

Range__________